Model Predictive Trajectory Set Control for a Proportional Directional Control Valve

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Model Predictive Trajectory Set Control for a Proportional Directional Control Valve

Artemi Makarow[1], Martin Keller[1], Christoph Rösmann[1], Torsten Bertram[1], Georg Schoppel[2], Ingo Glowatzky[2]

Abstract— **In practice, high quality control for mechatronic systems is often achieved by augmenting classical control architectures like PID controllers with numerous tailored nonlinear characteristic parameter curves and cascades. This complexity can be significantly reduced by utilizing advanced model predictive controllers (MPC). Furthermore, desired objectives like minimum control error and effort can be realized while explicitly adhering to state and control constraints. However, MPC is subject to iterative gradient-based online optimization algorithms which are computationally expensive. Hence, their application to mechatronic systems with fast dynamics is limited. It is worth mentioning that industrial systems often utilize low cost computational hardware. Accordingly, this contribution presents a model predictive trajectory set control (MPTSC) scheme that mimics a sub-optimal MPC by a sparse discretization of the control input domain. A comparative analysis with a linear quadratic regulator demonstrates its ability to provide a sufficiently high control performance compared to the optimal reference. Furthermore, the approach is experimentally evaluated on a proportional directional control valve with a sample rate of 10 kHz. In addition to its efficiency the implementation of MPTSC is less complex and error-prone in comparison to MPC which is a reasonable advantage especially in industrial applications.**

I. INTRODUCTION

For fast and precise control of mechatronic systems cascaded control architectures based on PID controllers are widespread in current industrial applications mainly due to their well-studied theory and their low computational demands [1]. In order to comply with application specific requirements classic PID controllers are extended with nonlinear characteristic curves for the integral and proportional amplification. Parameters are adapted as a function of the control error $e(t)$ and a desired rigorous controller performance is accomplished by increasing the number of parameters. However, the parametrization of such a complex controller is challenging. Either a process expert is needed for manually parameterizing the process or an automated hardware in the loop meta optimization can be utilized. For instance multi-criterial evolutionary computation techniques are applied in [2]. Nonetheless, the performance is limited since the controller does not incorporate any (at least vague) future knowledge about the possible evolution of the system behavior during runtime. State and control input constraints are only implicitly taken into account by choosing appropriate parameters in advance and thus the ability to operate close to the system's physical limits is waived. In addition, the simultaneous optimization of controller parameters and structure is promising but very time-consuming [3]. Another drawback is the possible over-fit of the controller structure and its parameters since the range of evaluated excitation signals is limited by practical reasons.

A suitable control concept for explicitly incorporating control and state constraints is model predictive control (MPC). Model predictive controllers repeatedly solve an optimal control problem over a moving finite horizon in each sampling interval [4]. Compared to PID control MPC predicts the future evolution of the system based on a dynamic model during runtime and achieves a higher control performance in terms of minimizing a user-defined but usually smooth objective function. However, solving such optimal control problems is computationally demanding. Consequently, researches started focusing on numerical efficient realizations and approximations of MPC during the last years. Diehl et al. present the multiple shooting approach as a direct solution to the optimal control problem in contrast to indirect methods based on calculus of variations [5]. Due to its tradeoff between sparsity and dimension of the optimization problem, multiple shooting turned out to converge faster than single shooting. Furthermore, computation time is reduced by the real-time iteration scheme which performs a single sequential quadratic programming iteration within each sampling interval and warm-starts from previous solutions [6]. In [7] the real-time iteration scheme is combined with first-order methods for embedded MPC applications. Other methods efficiently solve MPC problems with sparse interior-point-methods [8], [9], applicate projected gradients for real-time applications [10] or present code generation techniques for application specific and sparsity exploiting MPC solvers. Time-optimal MPC realizations for mechatronic systems are presented in [11], [12]. In the context of approximating MPC some publications deal with move-blocking strategies [13]. Hereby, the degrees of freedom in the control sequence are reduced. The reduction of the dimension of the optimization problem leads to faster but suboptimal solutions. In the extreme blocking case, the control variable is kept constant over the entire prediction horizon. For controlling mechatronic systems with sample rates above $f_\mathrm{s} = 1\,\mathrm{kHz}$ pre-computed piecewise state-dependent control laws can be designed [14]. The explicit MPC solves the parametric optimal control

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problem for all feasible states offline and hence the online tasks simplifies to the look-up operation. However, for large problem formulations the required memory storage and look-up time significantly increase due to the curse of dimensionality. Additionally, the offline computation must be repeated whenever parameters need to be changed. In the context of power electronics finite control set MPC (FCS-MPC) profits from the property of a limited number of switching states and thus can be executed quickly. The switching states of a converter represent the optimization variables [15], [16]. After the optimization the optimal switching states are directly set as active. A modulator which converts a continuous signal into corresponding switching states can be dropped. However, usually a one-step prediction horizon is used. A longer prediction horizon is not combined with an extreme move-blocking strategy since the converter is responsible for a rotating magnetic field.

In this paper we present a novel control concept suitable for fast mechatronic systems which accounts for the benefits of the classic MPC but also exhibits a low computational burden and guaranteed real-time capability. The number of computations in every control sample interval are well known in advance. The MPTSC mimics the move-blocking strategy and makes use of the advantages of FCS-MPC. The model predictive trajectory set control (MPTSC) uses a sparse discretization of the control input domain. Additionally, the inputs are kept constant over the prediction horizon. In every sampling interval a discrete set of trajectory candidates is generated from which the best one is selected for actual control. Hence, solving the optimal control problem with iterative gradient-based optimization techniques is dropped. The MPTSC approach is based on the idea of the model predictive trajectory set approach (MPTSA) developed for emergency maneuvers of vehicles in critical traffic situations in [17]. The MPTSA combines the planning and control of a collision avoidance trajectory in a single step. Due to the existence of the environment model, a pre-calculated reference signal is not necessary. In contrast to the MPTSA the MPTSC is meant to be a tracking controller and considers more general types of systems.

The contribution of this paper is to demonstrate that a sub-optimal MPC approach is able to satisfy the requirements for control quality of a mechatronic system while it reduces the complexity essentially. The proposed approach constitutes an application oriented solution. A hydraulic directional control valve is used as a mechatronic system to point out the applicability of the novel control concept. Such a valve is characterized by distinct nonlinearities and the fast dynamics require a sampling rate of $f_\mathrm{s} = 10\,\mathrm{kHz}$.

The next section describes the MPTSC in detail. In order to analyze the performance of the MPTSC approach, a comparison with the linear-quadratic regulator (LQR) is provided for a linear dynamic system in section II. A hydraulic direction control valve is described in section III which is later used for the experimental validation in section IV. Finally, section V summarizes the results and provides an outlook on further work.

II. MODEL PREDICTIVE TRAJECTORY SET CONTROL

For the reminder of this paper a single-input system is considered. The discrete state equations with p states and a single input are obtained by finite-differences and sample time ΔT:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{f}(\mathbf{x}_k, u_k)\,\Delta T, \quad \mathbf{x}_{k=0} = \mathbf{x}_0. \tag{1}$$

Hereby, $\mathbf{x}_k \in \mathbb{R}^p$ denotes the state vector at time instance k and $u_k \in \mathbb{R}$ the corresponding control input. Control inputs are further limited by the following box constraint:

$$u_\mathrm{min} \le u_k \le u_\mathrm{max}. \tag{2}$$

For the optimal control problem, the control input domain $\mathbb{R}$ is discretized such that $\mathbb{A} \subset \mathbb{R}$. In this paper an equidistant discretization with step size Δu is considered. The resulting control inputs are defined as follows:

$$u \in \mathbb{A} := \{u_\mathrm{min}, u_\mathrm{min} + \Delta u, u_\mathrm{min} + 2\,\Delta u, ..., u_\mathrm{max}\} \tag{3}$$

The (sub-)optimal control input u_k^* is obtained as follows:

$$u_k^* = \arg\min_{u \in \mathbb{A}} J_\mathrm{M}(\mathbf{x}_k, u) \tag{4}$$

subject to (1), (3) and the state box constraints

$$\mathbf{x}_\mathrm{min} \le \mathbf{x}_k \le \mathbf{x}_\mathrm{max}. \tag{5}$$

The implicit control law at time step k is now defined as $u_k = u_k^*$. A control horizon of $n_\mathrm{c} = 1$ is chosen such that the control variable is kept constant over the prediction horizon t_p. The optimal control problem in (4) is solved in the manner that within each sampling interval ΔT_s a set of trajectory candidates is predicted according to all possible controls $u \in \mathbb{A}$. The candidate with the least objective function value $J_\mathrm{M}(\cdot)$ is chosen and the corresponding input u_k^* is applied to the plant. Note, the requirements on $J_\mathrm{M}(\cdot)$ are fairly mild, since no gradient-based optimization is performed. Non-smooth cost functions might be utilized dependent on the application.

A. Simulative Analysis of the MPTSC – Setup

For the first evaluation of the control performance of the MPTSC a linear dynamic system is used for both prediction and plant simulation. Note, the linear model is chosen in preparation for the dynamic directional control valve characteristics presented in section III. The system is defined by the following differential equation:

$$\ddddot{x} = -a_2\dddot{x} - a_1\ddot{x} - a_0\dot{x} + b_0 u. \tag{6}$$

Transforming this equation into a state space representation with state vector $\mathbf{x} = [x_1, x_2, x_3, x_4]^\intercal = [x, \dot{x}, \ddot{x}, \dddot{x}]^\intercal$ results in a fourth order continuous-time system:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u) = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dddot{x} \\ -a_0\dot{x} - a_1\ddot{x} - a_2\dddot{x} + b_0 u \end{bmatrix}. \tag{7}$$

A plant without self-regulation is applied with intent. In case of state regulator's model mismatch the closed loop control might exhibit a non-zero offset characteristic. A common policy to achieve offset free tracking of the reference is to augment the system with a virtual integrator [18]. Moreover, for a sparse control input discretization the MPTSC might not be able to perfectly reach the reference respectively steady-state due to reduced degrees of freedom close the target state. On this occasion, an augmented integrator addresses these circumstances.

In the simulative analysis section, the model is numerically solved with the Euler method according to (1). The formulation accounts for a dedicated step size of $\Delta T_{\mathrm{e}} = 0.01\,\mathrm{s}$ for the future state prediction in comparison to the closed-loop sampling interval. Variable $\Delta T_{J_{\mathrm{M}}}$ denotes the time interval between predicted states $\hat{\mathbf{x}}_{k,l}(u) := \hat{\mathbf{x}}_l(\mathbf{x}_k, u)$ of the objective function. The index k indicates the runtime sampling and the index l indicates the steps during the prediction. For small time steps $\Delta T_{J_{\mathrm{M}}}$ numerical integration in objective function is performed by applying the explicit Euler method:

$$J_{\mathrm{M}}(\mathbf{x}, u) = \int\limits_{t=0}^{t=t_{\mathrm{p}}} \left((\mathbf{x} - \mathbf{x}_{\mathrm{f}})^{\intercal} \mathbf{Q}_{\mathrm{M}} (\mathbf{x} - \mathbf{x}_{\mathrm{f}}) + r_{\mathrm{M}} u^2\right) \mathrm{d}t \tag{8}$$

$$J_{\mathrm{M}}(\mathbf{x}_k, u) \approx \Delta T_{J_{\mathrm{M}}} \sum_{l=1}^{l=n_{\mathrm{p}}} \left((\hat{\mathbf{x}}_{k,l} - \mathbf{x}_{\mathrm{f}})^{\intercal} \mathbf{Q}_{\mathrm{M}} (\hat{\mathbf{x}}_{k,l} - \mathbf{x}_{\mathrm{f}}) + r_{\mathrm{M}} u^2\right).$$

with $\hat{\mathbf{x}}_{k,0} = \mathbf{x}_k$ and reference target state $\mathbf{x}_f$. In this contribution the step intervals are set to $\Delta T_{J_{\mathrm{M}}} = \Delta T_{\mathrm{e}} = 0.01\,\mathrm{s}$. $\mathbf{Q}_{\mathrm{M}}$ denotes at least a quadratic semi-definite weight matrix for the penalization of the state error $\mathbf{x} - \mathbf{x}_f$ and r_M is a scalar weight for penalizing the control effort.

The continuous-time linear-quadratic regulator (LQR) with an infinite horizon provides an optimal reference if the quadratic objective function

$$J_{\mathrm{L}} = \int\limits_{0}^{\infty} \left((\mathbf{x} - \mathbf{x}_f)^{\intercal} \mathbf{Q}_{\mathrm{L}} (\mathbf{x} - \mathbf{x}_f) + r_{\mathrm{L}} u^2\right) \mathrm{d}t \tag{9}$$

subject to the linear system dynamics $\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{b}\,u$ is used ($u \in \mathbb{R}$). The control law in analytical form is

$$u_{\mathrm{L}}^* = -\mathbf{k}\,(\mathbf{x} - \mathbf{x}_{\mathrm{f}}) \tag{10}$$

where $\mathbf{k}$ is given by

$$\mathbf{k} = {r_{\mathrm{L}}}^{-1} (\mathbf{b}^{\intercal} \mathbf{P}) \tag{11}$$

and the algebaric Riccati equation

$$\mathbf{A}^{\intercal} \mathbf{P} + \mathbf{P}\mathbf{A} - (\mathbf{P}\,\mathbf{b})\, {r_{\mathrm{L}}}^{-1} (\mathbf{b}^{\intercal} \mathbf{P}) + \mathbf{Q}_{\mathrm{L}} = \mathbf{0} \tag{12}$$

must be solved with respect to $\mathbf{P}$. The LQR does not support any state and control constraints and exhibits at least a positive semi-definite diagonal matrix $\mathbf{Q}_{\mathrm{L}}$ and $r_{\mathrm{L}} > 0$. The overall control task is the transition between the initial states $\mathbf{x}_0$ and the final states $\mathbf{x}_{\mathrm{f}} = [x_{\mathrm{ref}}, 0, 0, 0]^{\intercal}$. The first state x_{ref} is defined as a function of time to mimic a step sequence. Note, in the remainder of this contribution, the controllers are not aware of future switching modes in x_{ref}. Hence, they keep the sampled final state constant over the prediction horizon at each sampling interval. For a comparative analysis of the LQR and MPTSC in terms of control performance the following fit-criterion is applied:

$$fit = 100\,\% \left(1 - \frac{\|x_{1,\mathrm{LQR}} - x_{1,\mathrm{MPTSC}}\|}{\|x_{1,\mathrm{LQR}} - \overline{x}_{1,\mathrm{LQR}}\|}\right). \tag{13}$$

For the optimization of free parameters a genetic algorithm is utilized to maximize the fit-criterion for the given step sequence up to convergence [19]. The elements of the matrix $\mathbf{Q}_{\mathrm{M}}$, the scalar penalty r_{M} and the prediction horizon t_{p} are optimized. Since the control horizon is limited, in particular $n_{\mathrm{c}} = 1$, t_{p} has a greater influence on the closed loop performance. Assuming that just the weight $q_{\mathrm{M}_{11}}$ for the state error in x_1 is non-zero, a rather short prediction horizon leads to a fast dynamic performance since the trajectories cross the reference x_{ref} when the distance between it and the actual state x_1 is already small. The maximum opposite input value $u_{\max}$ or $u_{\min}$ is potentially too low to preserve the control variable from overshooting. A rather long prediction horizon leads to an over-damped dynamic performance since the reference is detected earlier by the crossing of trajectories.

B. Simulative Analysis of the MPTSC – Results

In the first scenario the state constraints (5) are omitted. The model parameters are $b_0 = 1$, $a_2 = 1.75$, $a_1 = 1.5$ and $a_0 = 1$. The parameters of the LQR, the elements of the diagonal matrix $\mathbf{Q}_{\mathrm{L}}$ and the scalar penalty r_{L} are tuned in the way that a high control performance is reached. In particular, control performance is measured in terms of common design parameters for mechatronic systems like rise time, settle time and overshoot characteristic. Simultaneously, a violation of the box constraint on the control variable in (2) with $|u_k| \leq 10^3$ is avoided. As long as no constraints are violated the LQR leads to an absolute reference for the control performance for the described setup. It is well-known from the theory of MPC that even a predictive controller with control horizon $n_c = 1$ is able to mimic an infinite horizon (and hence an LQR in the above case) if the state error weight is chosen according to the algebraic Riccati equation and some further preliminaries are satisfied [20]. As a reminder, the MPTSC further discretizes the control input domain.

To evaluate the performance of the MPTSC, the goal is to reach the optimal reference of the LQR as close as possible. The control sampling interval ΔT_{s} is equal to the interval $\Delta T_e = 0.01\,\mathrm{s}$. For the MPTSC the set of discrete control variables $\mathbb{A}$ contains 71 values with $\Delta u = 2{\cdot}10^3/71$. Figure 1 indicates the (suboptimal) performance of the MPTSC in comparison to the optimal LQR. For the given step sequence MPTSC can reproduce the LQR performance with a comparable characteristic with a value of $fit = 96.8\,\%$. It was known in advance that a suboptimal controller cannot reach the optimal controller performance exactly. However, it is close to the optimal solution with a set $\mathbb{A}$ of just 71 values. A slight oscillation occurs around the steady-state as a result of the sparse input discretization. This is due to

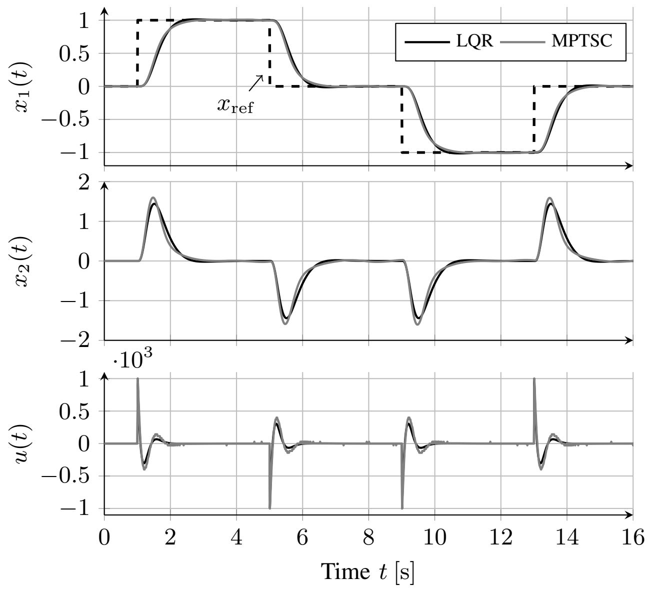

Fig. 1: Evaluation of the suboptimal closed loop performance of the MPTSC. The states x_3 and x_4 are omitted here.

the fact that the exact required input value for the steady-state is not in the input set $\mathbb{A}$. This effect is just noticeable in case of zero model mismatch and can be seen in the bottom plot. Small deflections around the zero input can be extracted. With an input set $\mathbb{A}$ of just 21 values the LQR can still be reproduced with a performance of $fit = 96.1\,\%$. However, the amplitude of the oscillation around the steady state increases slightly. Consequently, the number of values in set $\mathbb{A}$ must be chosen according to the desired application and available computational resources. For a more detailed illustration of the functionality of the MPTSC, the set of predicted trajectories of the first state x_1 is shown for multiple control sampling steps in Figure 2. With respect to the objective function, a single trajectory is selected by applying the minimum operator to the candidate set. For the sake of clearness, the prediction horizon is set longer as it is required and the set $\mathbb{A}$ contains only 11 discrete values. In the next scenario the box constraints $|u| \leq 10^3$, $|x_2| \leq 0.6$, and $|x_3| \leq 3$ are activated. Figure 3 demonstrates the capability of the MPTSC to handle box state constraints for the presented example system. The simulative analysis indicates promising control performances. In the following MPTSC is applicated to a real mechatronic system.

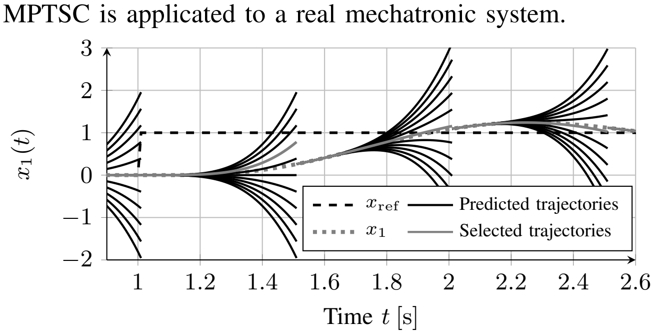

Fig. 2: Predicted trajectories for three control sample steps. The prediction horizon is set to $t_{\mathrm{p}} = 0.5\,\mathrm{s}$.

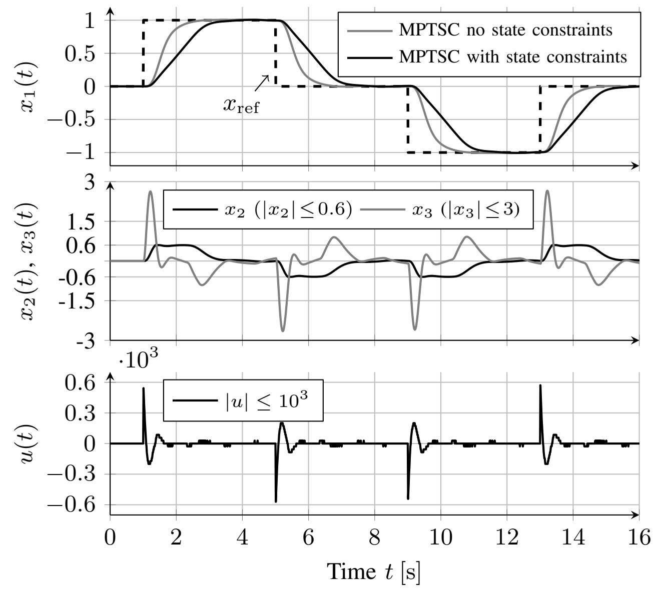

Fig. 3: Evaluation of the closed loop performance of the MPTSC with active box constraints. State x_4 is omitted here.

III. DIRECTIONAL CONTROL VALVE

In this paper a 4WRPEH6 type valve of Bosch Rexroth AG is utilized. Figure 4 illustrates a cross section of the valve. The function of the valve is to route a flow rate Q_{v} from the pressure port P to the ports A or B since hydraulic operated actuators may be connected to this ports. The oil flow is depending on the piston position and the supply pressure at port P. The feedback control problem of the valve is the accurate and fast positioning of the pistons position. The position is measured with an internal inductive sensor. A position indicated with $x_1 = \pm 100\,\%$ denotes a fully opened working port A or B. While one port is set as active a pressure relief occurs between the other port and the tank T. The positive movement direction is forced by the solenoid excited with the current, while the negative movement direction is forced by the spring. The performance of this fast acting mechatronic system is characterized by plenty of nonlinearities like magnetic hysteresis and dynamic friction. As it was already mentioned a cascaded control concept is established. The inner controller is a rather simple current controller with the measurable state i as a fast acting control variable. The position controller exhibits a high

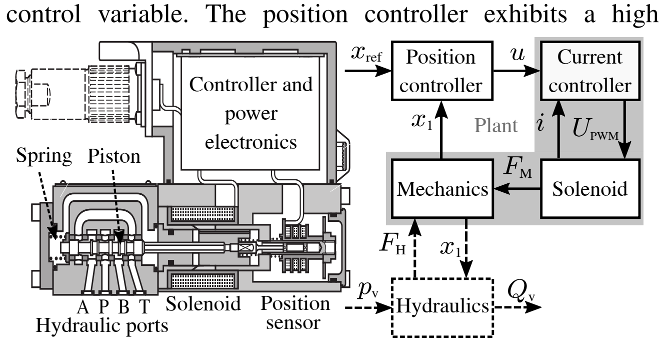

Fig. 4: Cross section of the hydraulic directional control valve. The block diagram illustrates the mechatronic character of the valve and the native cascaded control concept.

number of parameters and thus a high complexity in order to achieve the requirements for the closed loop performance. The native control concept is performed with a sampling rate of $f_\mathrm{s} = 10\,\mathrm{kHz}$. The outer position controller is replaced for further analyzes with the new MPTSC. Due to the fact that the rest of the control structure is preserved, the application effort for realizing this approach is rather low. Since it is not known in advance for which application the valve will be used, the goal is to find a robust set of controller parameters which provides sufficiently high performance for all operating scenarios. If the pressure and the flow rate were measurable states, the hydraulic interactions could be included in the dynamic model to gain higher functionality and parameters generalization. In this contribution a dynamic model is identified to map the input of the current controller u to the output of the position sensor x_1. The input is limited to the value range of $U = [-100\,\%, 100\,\%]$ according to (2).

IV. EXPERIMENTAL RESULTS

In section II the structure of the dynamic model for the directional control valve is already presented. At least a linear model of third order is required for an accurate prediction quality. In order to deal with steady-state offsets mentioned before, a virtual integrator is inserted. Instead of discretizing the value range of the real control variable the deviation of this control variable, which is limited to $\dot{U} = [-3{\cdot}10^4\,\%\,\mathrm{s}^{-1}, 3{\cdot}10^4\,\%\,\mathrm{s}^{-1}]$, is discretized. The states x_1, x_2 and x_3 are physically interpretable, namely they describe the position, the velocity and the acceleration of the movable part of the valve. A Luenberger observer is utilized based on the identified linear dynamic model. In addition to the measurable piston position x_1 the current i is also measurable and is used by the current controller. A prediction of the position on the basis of the current i needs a nonlinear model so that this measurable state does not lead to any improvement when using a linear model. It is obvious that a linear model cannot be accurate for all operating areas and respects all important physical properties. To do so, a complex model must be developed with a high simulation quality [21]. Just regarding the fact that the considered valve has an asymmetrical force effect. However, it has its advantages in terms of the real-time capability and thus allows the functionality of the MPTSC to be demonstrated in the experiment. The dynamic system in (7) can be analytically solved for a constant input so that a numerical integration method is not necessary for the prediction in every control sampling interval. At runtime the following algebraic equation has to be solved:

$$\hat{\mathbf{x}}_{k,l} = \Phi_l\,\mathbf{x}_k + \Gamma_l\,u, \quad \Phi_l \in \mathbb{R}^{p\times p}, \quad \Gamma_l \in \mathbb{R}^{p\times 1}. \tag{14}$$

Φ_l is the well-known transition matrix. The matrices Φ_l and Γ_l can be computed offline for the desired prediction time steps l. For the realization of the real-time performance with a control sample rate of $f_\mathrm{s} = 10\,\mathrm{kHz}$ the evaluation of the objective function is carried out for just a few points. Here, only five values are evaluated for a prediction horizon of $t_\mathrm{p} = 0.5\,T$ with $\Delta T_{J_\mathrm{M}} = 0.1\,T$. It is assumed that the approximation in (8) is still valid. The variable T indicates a normalized time. In the context of the optimization of the native PID control concept of the directional control valve more than one criterion is considered. The controller parameters are optimized to obtain a short rise time, a short settling time and a small overshoot. The parameters of the MPTSC are optimized using a multi-criteria evolutionary hardware in the loop optimization with a dominance-based genetic algorithm (NSGA-II) [19]. However, the small number and the mostly intuitive character of the parameters also allow a manual controller design. The control is performed with a real-time PC system setup (Matlab / Simulink Real-Time).

Practical investigations have shown that some additional objective parameters in (8) lead to a higher control performance. The first additional parameter ζ explicitly penalizes the predicted trajectories of the first state x_1 whenever they cross the reference x_ref. At runtime the matrix $\mathbf{Q}_\mathrm{M}$ is then weighted with ζ. It is similar to a soft constraint since there is still the possibility that a trajectory is chosen that crosses the reference x_ref. It has the advantage that the overall system performance is not slowed down as it is in the case of increasing the penalty $q_{\mathrm{M}_{22}}$ for the velocity state x_2. The second additional parameter λ is well-known as the forgetting factor or decay rate. Since a linear model leads to a non-neglectable model mismatch, it is reasonable to weight predicted future distant points in an exponentially descending manner. For the evaluation of five points on the prediction horizon the weighting vector has the form

$$\boldsymbol{\lambda} = [\lambda, \lambda^2, \lambda^3, \lambda^4, \lambda^5]^\intercal, \quad \lambda \leq 1. \tag{15}$$

The future distant points can be weighted lighter but they cannot be omitted since they support the closed loop stability. Parameters $q_{\mathrm{M}_{11}}$, $q_{\mathrm{M}_{22}}$, ζ, λ and t_p are optimized. The remaining parameters are set to zero. After the optimization a Pareto-optimal individual is chosen manually in dependence on the rise time, settling time and the overshoot behavior. The model parameters a_2, a_1, a_0 and b_0 are identified for different operating areas. Figure 5 indicates the closed loop performance of the real valve. Due to the low required displacement, a model parameter set can be found which represents both movement directions and leads to high performance. In contrast to the theoretical investigations in section II, the real control variable rather than it's deviation is shown. Additionally, Figure 5 shows the observer performance for the states x_2 and x_3. As the ground truth an offline zero-phase digital filtering of the measurable state x_1 is performed. The overall observer error is low but in the steady-state an oscillation due to the required high observer gain is visible. This fact leads to a recurring slightly false initialization in every control sampling interval. The suboptimal characteristic of the MPTSC as a result of the equidistant discretized input has no noticeable impact on the real closed loop performance. In this case, other intervening factors like the model mismatch or the observer error have a higher impact on the control quality. Figure 6 demonstrates another experiment which can be executed with a single linear dynamic model. Both experimental results are in the performance range of the native valve PID control concept.

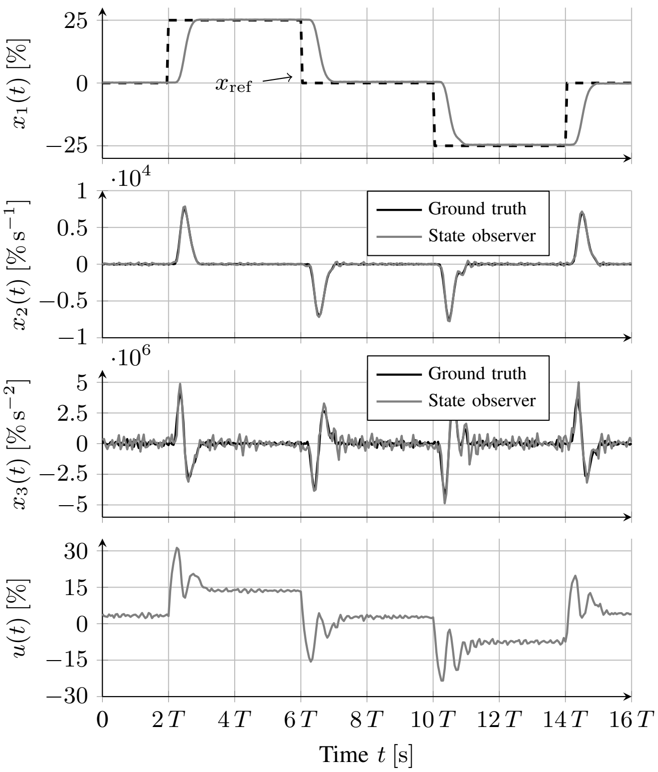

Fig. 5: Experimental closed loop performance results of the MPTSC for small displacements around the zero position.

V. CONCLUSIONS AND FUTURE WORK

This paper presents the model predictive trajectory set control. This control concept realizes a rough approximation of the classic MPC. However, it provides substantially high control performances in the closed loop, especially with the application to a directional control valve. Like the classic MPC it explicitly adheres to state and input constraints. Due to the sparse discretization of the control input domain, a gradient-based optimization algorithm is not required. This fact leads to considerable run-time and realization advantages. This paper demonstrates that the sub-optimal concept itself performs well and can already be used for the control task of a fast and nonlinear directional control valve.

Future work is concerned with the improvement of the dynamic model to predict the future process evolution and the observer for the hydraulic valve's states. It is conceivable to use multiple local linear models for the prediction. An adaptive discretization of the input at runtime can improve the control quality and further reduce the computational effort. Furthermore, a theoretical analysis and a wider comparison to MPC on different benchmark and mechatronic systems is of substantial interest.

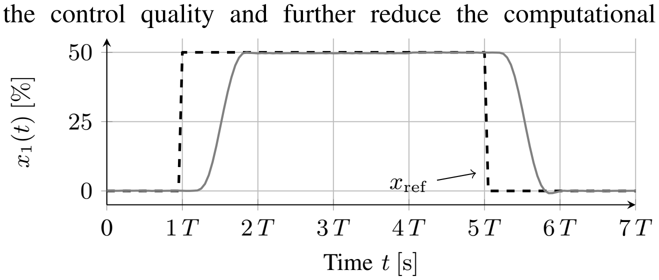

Fig. 6: Experimental closed loop performance results of the MPTSC for a high displacement from the zero position.

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